A NOTE ON INDUCED GRID NOISE AND NOISE FACTOR*

by

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SUMMARY

Comparison between theory and experiment on induced grid noise is discussed in relation to other published work. Calculation of the noise factor of a common-cathode triode circuit is carried out by a novel system of valve circuit analysis which ensures that transit time effects are inherent in the results. The results confirm the possibility of reducing the noise factor by "de-tuning" the input circuit with or without neutralization.

1.0. Induced Grid Noise

1.1. In a recent paper,[1] experimental results were quoted which suggested that an approximate relation exists between the induced grid noise in a triode with common cathode connections and the electronic part of the valve input capacitance. Also, by assuming that the valve acts, both for signals and for valve noise, as an ordinary complex circuit element, the relation

$$\overline{di_g^2} = \left|\frac{j\omega c_t}{g_m}\right|^2 . \overline{di^2} \quad \ldots\ldots\ldots (1)$$

was deduced by pure circuit analysis. (In this, $\overline{di_g^2}$ is the mean square induced grid noise current and $\overline{di^2}$ is the space-charge smoothed shot current, for the frequency response range f to $f + df$. c_t is the electronic part of the input capacitance).

In the same work, theoretical results derived elsewhere[2] were also quoted and, as was stated, bore little resemblance to the observed results.

1.2. The primary aim of the present note is to show that other existing theory, properly applied, bears a much closer resemblance to the same experimental results than the comparison quoted above would lead one to believe. In a paper recently published[3] the writer gave an expression (eqn. 24) connecting the correlated fluctuation currents in the cathode-grid and the grid-anode meshes of a basic triode circuit. To an accuracy including the first power in $\omega\tau$, this expression also follows from a combination of results of earlier works by others.[4,5] The induced grid noise current is given by the difference between the two mesh currents, and may be written

$$\delta i_g = \left(\frac{1}{3} j\omega\tau_1 + \frac{2}{3} j\omega\tau_2\right) . \delta i_1$$

or

$$\overline{di_g^2} = \left|\frac{1}{3} j\omega\tau_1 + \frac{2}{3} j\omega\tau_2\right|^2 . \overline{di^2} \quad \ldots\ldots\ldots (2)$$

(In this, τ_1 is the transit time in the cathode-grid space, and τ_2 is the transit time in the grid-anode space.)

This is more conveniently expressed :

$$\overline{di_g^2} = \left(\frac{1}{3}\omega\tau_1\right)^2 . \left(1 + 2\frac{\tau_2}{\tau_1}\right)^2 . \overline{di^2} \ldots (3)$$

from which, with the usual expressions for τ_1 and τ_2, the value of $\overline{di_g^2}$ can readily be calculated.

Thus, from the well-known expressions for τ_1 and τ_2 :

$$\frac{\tau_2}{\tau_1} = \frac{2d_2 V_c^{1/2}}{3d_1(V_c^{1/2}+V_a^{1/2})}$$

where V_c is the effective control voltage in the grid plane, calculated from the anode current I_a using the 3/2 power law. d_1 and d_2 are the cathode-grid and the grid-anode spacings. For $\overline{di^2}$ it is most expedient to use Rack's expression $4k\ (0{\cdot}644\ \theta_c)\ g_m\ .\ df$. By these means relative values of $\overline{i_g^2}$ have been calculated as a function of I_a and have been plotted as curves in Fig. 1, together with the curve obtained from the relation[2]

$$\overline{di_g^2} = \left(\frac{1}{5}\omega\tau_1\right)^2 . \overline{di^2} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The curves are based on a theoretical triode with $d_1 = 0{\cdot}1$ mm, $d_2 = 0{\cdot}3$ mm, and a cathode area of $0{\cdot}25\ \text{cm}^2$, obeying the 3/2 power law for which $g_m \propto I_a^{1/3}$.

1.3. On comparison with the experimental curves,[1] using the curve derived from eqn. (4) as a comparison standard, it is seen that values of $\overline{i_g^2}$ given by eqn. (3) are not inconsistent with experimental values in the region 6 to 10 mA

* Manuscript received September 22nd, 1950.
U.D.C. No. 621.396.822:621.396.6.

anode current. The marked discrepancy between the slopes of the theoretical and experimental curves is probably a result of the true values of τ_1 and g_m as a function of I_a differing from the "3/2 power law" values at lower currents on account of *Inselbildung*, undoubtedly present in the experimental valves considered.

1.4. The expression for the electronic part of the input capacitance derived from the Benham-Llewellyn theory, assuming zero initial emission velocity, is

$$\omega c_t = g_m\left(\frac{1}{6}\omega\tau_1 + \frac{2}{3}\omega\tau_2\right) \quad \ldots\ldots\ldots\ldots(5)$$

to an accuracy including the first power in $\omega\tau$. The use of this in formula (1) results in the curves drawn as broken lines in Fig. 1. It is evident that the values are low compared with values obtained[1] from the measured values of c_t.

1.5. Values of induced grid noise deduced from formula (1) by using formula (5) for c_t differ from values deduced directly from formula (3), whereas according to the method[1] of deriving formula (1), they should be alike. The reason is evident from a comparison of (2) and (5), from which it is seen that the valve acts as a circuit element for internally generated noise different from that for applied signals. Therefore, according to the school of thought followed here,[4,5] formula (1) is at best only approximately valid. It is not surprising, then, that the experimental curves, and the curves calculated from (1) by using measured values of c_t, do not agree. The appeal to experimental error to explain the difference between the two sets of results would then be unnecessary.

1.6. Formula (4) is widely at variance with formula (2) or (3) adopted here. Firstly, this results from the neglect of the grid-anode transit time τ_2 in the derivation[2] of (4). It is worthy of note that τ_2 was also neglected in other earlier work,[4] but experimental verification was made through a formula in terms of the electronic damping, which also depends on τ_2 in a similar manner. Secondly, the numerical coefficient of $\omega\tau_1$ in (4) is 1/5, whereas in equation (3) it is 1/3. In the writer's opinion the coefficient 1/3 is correct.

2.0. The Noise Factor of the Common Cathode Circuit

2.1. In a recent note[6] the result of applying formula (1) to the noise factor estimation of a

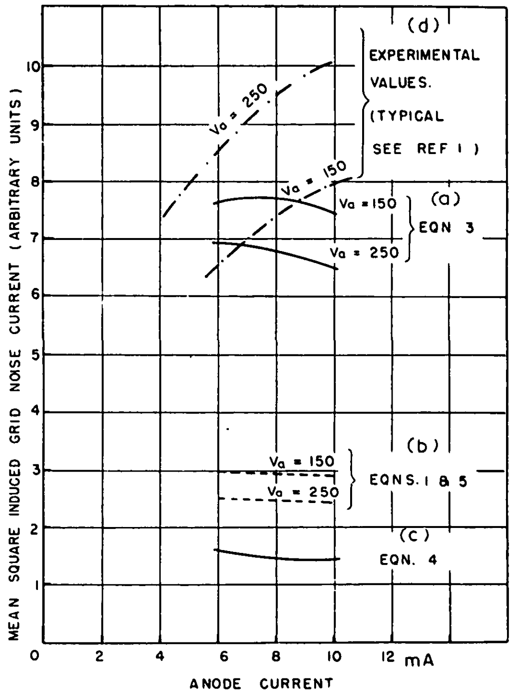

Fig. 1.—Values of induced grid noise: (a) Calculated on the theory of references 3, 4 and 5; (b) Calculated on the theory of reference 1 with theoretical values of c_t; (c) Calculated on the theory of reference 2; (d) Typical experimental values. (Reference 1.)

Note.—The values of Va on the experimental curves are in error and should be interchanged.

common-cathode triode circuit has been stated to be that, among other things, the noise factor is a minimum when the input circuit is "detuned" approximately by c_t. The idea of improving the noise factor by detuning the input circuit was also described some time ago by others.[7]

It will be shown here that this effect also follows from a treatment of the problem based on the theory[3] leading to formula (3), using a novel form of "operational model" of the valve as a circuit element. This model is illustrated in Fig. 2, in which the valve *per se* is regarded as a passive circuit element described by a set of simultaneous linear equations between the *mesh* current associated with each adjacent pair of electrodes and a small-signal voltage applied between each electrode and a common external point. This model is a development by the writer from one in which the valve was regarded as a multi-terminal network.[8] The chief advantage of the proposed model is that transit time effects come naturally into the calculations, and neither have to be grafted on to the system nor complicate it out of all proportion to its utility.

The relations defining the "circuit element" of Fig. 2 are :

$$\left.\begin{aligned} i_1 &= \left[y_1\left(1+\frac{1}{\mu}\right)+b_1\right]e_1+(y_1+b_1)e_2 \\ &\quad +\frac{y_1}{\mu}e_3+\delta i_1 \\ i_2 &= y_2\left(1+\frac{1}{\mu}\right)e_1+(y_2-b_2)e_2 \\ &\quad +\left(\frac{y_2}{\mu}+b_2\right)e_3+\delta i_2 \\ i_3 &= b_3e_1+b_3e_3. \end{aligned}\right\} \quad (6)$$

(i_3 is subsidiary, and often negligible).

(Here, y_1 is the electronic admittance of space I, y_2 is the electronic transadmittance of the current in space II relative to the voltage across space I, b_1 and b_2 are the "cold" susceptances across spaces I and II, respectively, and b_3 is the direct susceptance across both spaces. The b's are essentially $j\omega c$'s where c is the cold capacitance between two electrodes, but the b's may also include the effect of external reactance used for neutralizing. The current components δi and δi_2 are the correlated noise current components induced in spaces I and II respectively.)

2.2. While it is hoped to give a fuller description of this circuit model of the valve and its application at a later date, an illustration of the method provided by application to part of the circuit for the measurement of induced grid noise is of interest here.

Referring to Fig. 2, the anode and cathode are effectively grounded (i.e. $e_1 = e_3 = 0$) and there is an external admittance Y connected between grid and earth (i.e. $e_2 = -(i_1-i_2)/Y$). Solving the first two equations of (6) for i_1-i_2 results in :

$$i_1 - i_2 = \frac{Y(\delta i_1 - \delta i_2)}{Y+(y_1-y_2)+(b_1+b_2)}$$

$$\text{or} \quad \overline{di^2} = \frac{|Y|^2 \cdot |\delta i_1 - \delta i_2|^2}{|Y+(y_1-y_2)+(b_1+b_2)|^2} \quad ..(7)$$

in which $b_1+b_2 = j\omega(c_1+c_2)$, is the mean square noise current in Y.

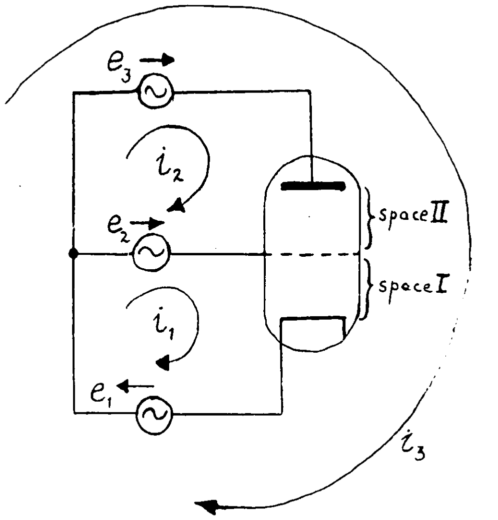

Fig. 2.—Basic triode circuit treating the valve as a passive circuit element. i_1, i_2 and i_3 are mesh currents, and e_1, e_2 and e_3 are ideal zero impedance small-signal generators.

The input damping and the electronic increase in capacitance are naturally included in the real and imaginary parts, respectively, of $y_1 - y_2$. This is better appreciated when it is noted that the values of y_1 and y_2, to the first power in $\omega\tau$, each has the magnitude, g_m, while y_1 has the lagging angle $1/5\ \omega\tau_1$ and y_2 has the lagging angle $11/30\ \omega\tau_1 + 2/3\ \omega\tau_2$.

Apart from this, the solution is the trivial one of division of current between two parallel admittances. The value of $|\delta i_1 - \delta i_2|^2$ is given by eqn. (2).

2.3. For the calculation of the noise factor N of a common-cathode triode circuit, the arrangement shown in Fig. 3 is considered. For present purposes, the noise of the input coupling circuit between the source and the valve electrodes is disregarded, thus making N the noise factor of the valve electrodes alone. There is an input load admittance $Y = 1/R_s + jB$ where the resistive part R_s is regarded as the source resistance at room temperature θ_0, and there is an anode load impedance $Z = R_L + jX_L$. The noise of R_L is, by convention, associated with the next stage. The noise factor is given by

$$N = 1 + \frac{\overline{i_{2v}^2}}{\overline{i_{2o}^2}}$$

where $\overline{i_{2v}^2} \,.\, R_L$ is the noise power in Z due to the valve alone, and $\overline{i_{2o}^2} \,.\, R_L$ is the noise power in Z due to the source noise across Y when the valve is ideally "silent." It is important to note that noise currents which are not correlated have to be dealt with separately, the results being combined by adding the mean square values. Therefore there are two problems here, one to evaluate $\overline{i_{2v}^2}$ and the other to evaluate $\overline{i_{2o}^2}$.

In equations (6) we have

$e_1 = 0$, $e_2 = -(i_1 - i_2)/Y$ and $e_3 = -Zi_2$.

To evaluate i_{2v}, put these values in (6), neglect i_3 (which in effect shunts Z and is thus trivial) and solve for i_2.

The result is :

$$i_{2v} = \delta i_2 \left\{ Y + \left(y_1 - y_2 \frac{\delta i_1}{\delta i_2} \right) + \left(b_1 + b_2 \frac{\delta i_1}{\delta i_2} \right) \right\} \Delta^{-1} \quad \ldots\ldots(8)$$

where

$$\Delta = Y + (y_1 - y_2) + (b_1 + b_2) + \text{terms in } Z.$$

To evaluate i_{2o}, we have to set $\delta i_1 = \delta i_2 = 0$ to represent an ideally silent valve, and put $e_2 = \delta i_o/Y - (i_1 - i_2)/Y$ where δi_o is the noise current in Y due to the source resistance R_s. Such a state is effected without re-solving (6) by putting $\delta i_1 = \delta i_o\,(y_1 + b_1)/Y$ and $\delta i_2 = \delta i_o\,(y_2 - b_2)/Y$ in eqn. (8). It follows that :

$$i_{2o} = \delta i_o (y_2 - b_2) \,.\, \Delta^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

and from this and eqn. (8) follows :

$$N = 1 + \frac{|\delta i_2|^2}{|\delta i_o|^2} \cdot \frac{\left| Y + \left(y_1 - y_2 \frac{\delta i_1}{\delta i_2} \right) + \left(b_1 + b_2 \frac{\delta i_1}{\delta i_2} \right) \right|^2}{|y_2 - b_2|^2} \quad \ldots\ldots\ldots\ldots(10)$$

The first factor may be resolved further by noting that $|\delta i_o|^2 = 4k\theta_o \,.\, df/R_s$, and by defining[9] the equivalent noise resistance R_n of the valve by $|\delta i_2|^2 = 4k\theta_o R_n |y_2|^2 \,.\, df$ (imperceptibly different from the value usually measured) which makes the first factor in (10) :

$$|\delta i_2|^2/|\delta i_o|^2 = R_n R_s \,.\, |y_2|^2.$$

2.4. Let us suppose that b_2 (normally $j\omega c_2$) is neutralized by parallel tuning, giving $b_2 = 0$ with a possible residual conductance which will be neglected. Setting $b_1 = j\omega c_1$ and $Y = 1/R_s + jB$, we have

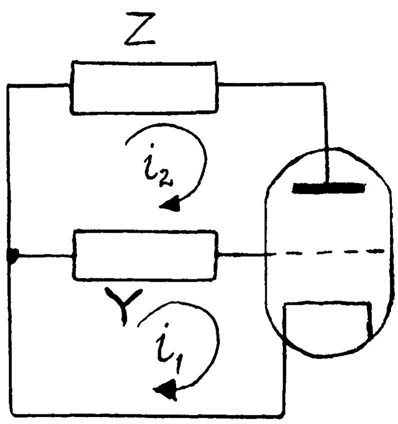

Fig. 3.—Common-cathode triode circuit with input circuit admittance Y and output circuit impedance Z.

$$N = 1 + R_n R_s \left| \frac{1}{R_s} + jB + \left(y_1 - y_2 \frac{\delta i_1}{\delta i_2} \right) + j\omega c_1 \right|^2 \quad \ldots\ldots\ldots\ldots(11)$$

The term $(y_1 - y_2 \,.\, \delta i_1/\delta i_2)$ has been discussed elsewhere,[9, 3] and by taking the first order values of y and δi (see ref. 3, para. 4.4.), is equal to $-1/6\ j\omega\tau_1\ g_m$. The first and second order values give also the real part $1/R_o$, believed to be equal to $11/90 \,.\, g_m \omega^2 \tau_1^2$. Therefore, if the input circuit is tuned to make

$$-B = \omega c_1 - \frac{1}{6}\omega\tau_1 g_m \quad \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

then (11) becomes simply

$$N = 1 + R_n R_s \left(\frac{1}{R_s} + \frac{1}{R_o} \right)^2 \quad \ldots\ldots\ldots(13)$$

and this represents the minimum value of N.

With signal tuning to obtain a maximum gain in a neutralized state ($b_2 = 0$), we have :

$$-B = \omega c_1 + \frac{1}{6}\omega\tau_1 g_m + \frac{2}{3}\omega\tau_2 g_m \quad \ldots\ldots(14)$$

and (11) becomes, on evaluating the modulus squared,

$$N = 1 + R_n R_s \left\{ \left(\frac{1}{R_s} + \frac{1}{R_o} \right)^2 + \left(\frac{1}{3}\omega\tau_1 + \frac{2}{3}\omega\tau_2 \right)^2 g_m^2 \right\} \quad \ldots\ldots(15)$$

2.5. Comparison of eqns. (12) and (14) shows that the susceptance difference between noise-tuning and gain-tuning of the input circuit is $(1/3\,\omega\tau_1 + 2/3\,\omega\tau_2) g_m$, which is precisely the value of the " ωc_t " which would have to be put in formula (1) to obtain the value of induced grid noise given by eqn. (2). It is not, however, the theoretical value of ωc_t, the electronic part of the input capacitance, as given by eqn. (5). Therefore, the " detuning " required to obtain a minimum noise factor is only approximately the electronic part of the input capacitance.

2.6. If the noise generated by the input coupling circuit, including the cathode coating losses, is accounted for by the noise of a shunt resistor R at a noise temperature $\lambda\theta_o$, then with noise-tuning (13) becomes :

$$N = 1 + \frac{\lambda R_s}{R} + R_n R_s \left(\frac{1}{R_s} + \frac{1}{R_o} + \frac{1}{R}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots(16)$$

if the noise due to the neutralizing circuit is disregarded. This relation is exactly the same as that for the grounded grid triode with noise-tuning.[9]

It is interesting to note that in (13) or (16) there is no contribution of noise from a " noisy " input damping as was suggested by earlier work under the name of " induced grid noise," but only the passive damping of an equivalent shunt resistor R_o. Even with gain-tuning, there is only additional passive damping, as shown by eqn. (15). Any " noisy " damping would involve a term outside the term in $R_n R_s$, although admittedly the term in $\omega\tau$ under $R_n R_s$ in (15) may be placed outside the large brackets and shown to be *approximately* equal to 5 R_s/R_t, where $1/R_t$ is the transit-time input damping (i.e. the real part of $y_1 - y_2$). The older theory therefore gives approximately correct results when gain-tuning adjustment is made to the input circuit.

Although induced grid noise can be measured artificially, it does not appear as such in an amplifier circuit, in so far as it is correlated to the shot noise. Only a non-correlated part, usually very small, would be an active noise source. These considerations show that *ad hoc* assumptions, such as ascribing a noise temperature to an input damping resistor grafted on to the system, can be misleading.

2.7. Returning to eqn. (10), it is evident that the noise factor is quite independent of the anode load Z. (Although the gain is, of course, dependent on Z.) Secondly, the denominator $|y_2 - b_2|^2$ is approximately equal to $|g_m - j\omega c_2|^2 = (g_m^2 + \omega^2 c_2^2)$. Also, when b_2 is capacitative, the term $b_2 \delta i_1/\delta i_2$ in the numerator contributes a negative conductance which reduces the total conductance in the numerator. Then, provided the tuning of the input is adjusted to cancel the susceptive part due to the valve, it follows that adjustment of b_2 to partial or no neutralization will reduce the noise factor.

This reduction of noise by feedback between anode and grid has been noted experimentally and recorded elsewhere.[10] Questions of satisfactory gain and stability, however, remain.

2.8. The considerations in section 2 of this note are limited to the case in which the pass-band of subsequent stages of amplification is narrow compared with the bandwidth of the input circuit. With a wide pass-band of subsequent amplification, a modified noise factor analysis would be necessary.

3.0. References

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